Graph Signal Processing and Applications

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1 Introduction

2 Wavelet Transforms on Arbitrary Graphs

3 Applications

4 Conclusions
Graphs provide a flexible model to represent many datasets:

- Examples in Euclidean domains

(a) Computer graphics
(b) Wireless sensor networks
(c) image - graphs
Motivation

- Examples in non-Euclidean settings

(a) Social Networks, (b) Finite State Machines (FSM)

Graphs can capture complex relational characteristics (e.g., spatial, topological).
Graph Signal Processing?

- **Graph**: Assume fixed

- **Signal**: set of scalars associated to graph vertices

- Define familiar notions: frequency, sampling, transforms, etc

- Use these for compression, denoising, interpolation, etc
Examples

- **Sensor network**
  - Relative positions of sensors (kNN), temperature
  - Does temperature vary smoothly?

- **Social network**
  - Friendship relationship, age
  - Are friends of similar age?

- **Images**
  - Pixel positions and similarity, pixel values
  - Discontinuities and smoothness
What do we know about transformations on Graphs?

$A$ and $D$: adjacency and degree matrices, $L = D - A$: graph Laplacian

$L$ can be interpreted as a local (high-pass) operation on this graph

Circulant matrix – Eigenvectors: DFT
Can we do similar things on more complex graphs?
Can we do similar things on more complex graphs?

Yes! But things get more complicated
A is no longer circulant – no DFT in general, but...

- Polynomials of $L = D - A$ or $A$ are local operators
- There will be a frequency interpretation: eigenvectors of $L$. 
What makes these “graph transforms”? 

Graph-based shift invariance – Operator is the same, local variations captured by $A$ or $L$. 

$H = L = D - A$ 

This can be generalized: 

$H = L - 1 \sum_{k=0}^{\alpha} L^k$ or $H = L - 1 \sum_{k=0}^{\alpha} A^k$ 

Or alternatively, based on Graph Fourier Transform
What makes these “graph transforms”? 

- Graph-based shift invariance – Operator is the same, local variations captured by $\mathbf{A}$ or $\mathbf{L}$.

\[
\mathbf{H} = \mathbf{L} = \mathbf{D} - \mathbf{A}
\]
What makes these “graph transforms”?

- Graph-based shift invariance – Operator is the same, local variations captured by $A$ or $L$.

$$H = L = D - A$$

- This can be generalized:

$$H = \sum_{k=0}^{L-1} \alpha_k L^k \quad \text{or} \quad H = \sum_{k=0}^{L-1} \alpha_k A^k$$

- Or alternatively, based on Graph Fourier Transform
Localized linear operations on graphs using polynomials of $A$ or $L$.
Frequency interpretation is possible for eigenvectors of $A$ or $L$.
A great deal depends on the topology of the graph.

In what follows we consider mostly undirected graphs without self loops and use $L$.
[Shuman, Narang, Frossard, Ortega, Vandergheysnt, SPM’2013]
Other approaches are possible based on $A$
[Sandryhaila and Moura 2013]
Research Goals

- Extend signal processing methods to arbitrary graphs
  - Downsampling, graph-frequency localization, multiresolution, wavelets, interpolation

Outcomes
- Work with massive graph-datasets: localized “frequency” analysis
- Novel insights about traditional applications (image/video processing)
- New applications

This talk
- Graph Signal Processing – intro
- Graph Filterbank design
- Applications
  - Depth image coding
  - Semi-supervised learning (2nd talk!)
Graphs 101

- Graph $G = (V, E, w)$.
- Adjacency matrix $A$
- Degree matrix $D = \text{diag}\{d_i\}$
- Laplacian matrix $L = D - A$.
- Normalized Laplacian matrix $\mathcal{L} = D^{-1/2}L D^{-1/2}$
- Graph Signal $f = \{f(1), f(2), ..., f(N)\}$

Assumptions:
1. Undirected graphs without self loops.
2. Scalar sample values
Spectrum of Graphs

- Graph Laplacian Matrix $\mathbf{L} = \mathbf{D} - \mathbf{A} = \mathbf{U}\Lambda\mathbf{U}'$

- Eigen-vectors of $\mathbf{L}$: $\mathbf{U} = \{\mathbf{u}_k\}_{k=1:N}$

- Eigen-values of $\mathbf{L}$: $\text{diag}\{\Lambda\} = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$

- Eigen-pair system $\{(\lambda_k, \mathbf{u}_k)\}$ provides Fourier-like interpretation — Graph Fourier Transform (GFT)
Graph Frequencies

(a) $\omega = \pi/4 \times 0$

(b) $\omega = \pi/4 \times 1$

(c) $\omega = \pi/4 \times 4$

(d) $\omega = \pi/4 \times 7$

DCT basis for regular signals

(a) $\lambda = 0.00$

(b) $\lambda = 0.04$

(c) $\lambda = 1.20$

(d) $\lambda = 1.55$

Eigenvectors of an arbitrary graph
Eigenvectors of graph Laplacian

(a) $\lambda = 0.00$  
(b) $\lambda = 0.04$  
(c) $\lambda = 0.20$  
(d) $\lambda = 0.40$  
(e) $\lambda = 1.20$  
(f) $\lambda = 1.49$
Graph Transforms

- Desirable properties
  - Invertible
  - Critically sampled
  - Orthogonal
  - Localized in graph (space) and graph spectrum (frequency)

- Local Linear Transform
- Can we define Graph Wavelets?
Next Section

1. Introduction

2. Wavelet Transforms on Arbitrary Graphs

3. Applications

4. Conclusions
Discrete Wavelet Transforms in 2 slides – 1

(d) (e)

From Vetterli and Kovacevic, Wavelets and Subband Coding, '95
Note: Filters have some frequency and space localization

From Vetterli and Kovacevic, [Ding'07]
Prior Work – Spectral Graph Transforms

- Designed in the spectral domain of the graph. Examples:
  - Diffusion Wavelets [Coifman and Maggioni 2006]
  - Spectral Wavelets on Graphs [Hammond et al. 2011]

- Spectral Wavelet transforms [Hammond et al. 2011]:
  Design spectral kernels: \( h(\lambda) : \sigma(G) \rightarrow \mathbb{R} \).

\[
T_h = h(\mathcal{L}) = \mathbf{U} h(\Lambda) \mathbf{U}^t
\]

where
\[
h(\Lambda) = \text{diag}\{h(\lambda_i)\}
Spectral Graph Transforms Cont’d

- Output Coefficients:

$$w_f = T_h f = \sum_{\lambda \in \sigma(G)} h(\lambda) \bar{f}(\lambda) u_\lambda$$

- Polynomial kernel approximation:

$$h(\lambda) \approx \sum_{k=0}^{K} a_k \lambda^k$$

$$T_h \approx \sum_{k=0}^{K} a_k L^k$$

$K$-hop localized: no spectral decomposition required.
Graph Filterbank Designs

- Formulation of critically sampled graph filterbank design problem
- Design filters using spectral techniques [Hammond et al. 2009].
- Orthogonal (not compactly supported) [IEEE TSP June 2012]
- Bi-Orthogonal (compactly supported) [IEEE TSP Oct 2013]
Downsampling/Upsampling in Graphs

Downsampling-upsampling operation:

- **Regular Signals:**
  \[ f_{du}(n) = \begin{cases} 
  f(n) & \text{if } n = 2m \\
  0 & \text{if } n = 2m + 1 
  \end{cases} \]

- **Graph signals:**
  \[ f_{du}(n) = \begin{cases} 
  f(n) & \text{if } n \in S \\
  0 & \text{if } n \notin S 
  \end{cases} \]
  for some set \( S \).

- For regular signals DU by 2 operation is equivalent to
  \[ F_{du}(e^{j\omega}) = \frac{1}{2}(F(e^{j\omega}) + F(e^{-j\omega})) \]
  in the DFT domain.

- What is the DU by 2 for graph signals in GFT domain?
Downsampling in Graphs

- Define $J_\beta = J_{\beta H} = \text{diag}\{\beta_H(n)\}$.
- In vector form:

$$f_{du} = \frac{1}{2}(f + J_\beta f) = \frac{1}{2}(f + \tilde{f})$$
Define $J_\beta = J_{\beta H} = \text{diag}\{\beta_H(n)\}$.

In vector form:

$$f_{du} = \frac{1}{2}(f + J_\beta f) = \frac{1}{2}(f + \tilde{f})$$

**Spectral Folding** [4]: For a bipartite graph $\tilde{f}(\lambda) = f(2 - \lambda)$. 
Downsampling in Graphs

- Define $J_\beta = J_{\beta_H} = \text{diag}\{\beta_H(n)\}$.
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- **Spectral Folding** [4]: For a bipartite graph $\tilde{f}(\lambda) = f(2 - \lambda)$. 
Graph filterbanks

- Filters designed in spectral domain (as [Hammond et al, 2009])
- Analysis:
  - $h_i(\lambda) : \mathbb{R} \rightarrow \mathbb{R}$ for $i = 0, 1$
  - $H_i = h_i(L) = U h_i(\Lambda) U^t$
- Synthesis:
  - $g_i(\lambda) : \mathbb{R} \rightarrow \mathbb{R}$
  - $G_i = g_i(L)$
Wavelet filterbanks on bipartite graphs

• **Aliasing Cancellation** $\Rightarrow B = 0$ if for all $\lambda \in \sigma(G)$:

$$B(\lambda) = g_1(\lambda)h_1(2 - \lambda) - g_0(\lambda)h_0(2 - \lambda) = 0$$
Wavelet filterbanks on bipartite graphs

- **Aliasing Cancellation** ⇒ $B = 0$ if for all $\lambda \in \sigma(G)$:

  \[ B(\lambda) = g_1(\lambda)h_1(2 - \lambda) - g_0(\lambda)h_0(2 - \lambda) = 0 \]

- **Perfect Reconstruction** ⇒ $A = cI$ if for all $\lambda \in \sigma(G)$:

  \[ A(\lambda) = g_1(\lambda)h_1(\lambda) + g_0(\lambda)h_0(\lambda) = c \]
GraphBior design

- Analogous to CDF wavelet Filters [Narang and Ortega, IEEE TSP, 2013]
GraphBior design

- Analogous to CDF wavelet Filters [Narang and Ortega, IEEE TSP, 2013]
- Choose kernels, s.t.,

\[
\begin{align*}
 h_0(\lambda) &= g_1(2 - \lambda) \\
 g_0(\lambda) &= h_1(2 - \lambda),
\end{align*}
\]

for aliasing cancellation \((B = 0)\).
GraphBior design

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  g_0(\lambda) &= h_1(2 - \lambda),
\end{align*}
\]

for aliasing cancellation (\(B = 0\)).

- The PR condition (\(A = 0\)) becomes:

\[
\underbrace{h_1(\lambda)g_1(\lambda)}_{p(\lambda)} + \underbrace{h_1(2 - \lambda)g_1(2 - \lambda)}_{p(2-\lambda)} = c
\]
GraphBior design

- Analogous to CDF wavelet Filters [Narang and Ortega, IEEE TSP, 2013]
- Choose kernels, s.t.,

\[ h_0(\lambda) = g_1(2 - \lambda) \]
\[ g_0(\lambda) = h_1(2 - \lambda), \]

for aliasing cancellation \((B = 0)\).
- The PR condition \((A = 0)\) becomes:

\[
\frac{h_1(\lambda)g_1(\lambda)}{p(\lambda)} + \frac{h_1(2 - \lambda)g_1(2 - \lambda)}{p(2-\lambda)} = c
\]

- Design \(p(\lambda)\) as a “maximally flat” polynomial and factorize into \(h_1(\lambda), g_1(\lambda)\) terms. Exact reconstruction with polynomial filter (compact support).
Bipartite Subgraph Decomposition

- But not all graphs are bipartite...
Bipartite Subgraph Decomposition

- But not all graphs are bipartite...
- Solution: “Iteratively” decompose non-bipartite graph $G$ into $K$ bipartite subgraphs:
  - each subgraph covers the same vertex set.
  - each edge in $G$ belongs to exactly one bipartite graph.
Bipartite Subgraph Decomposition

- Example of a 2-dimensional ($K = 2$) decomposition:

$G_0$
Example of a 2-dimensional \((K = 2)\) decomposition:
Example of a 2-dimensional \((K = 2)\) decomposition:
Bipartite Subgraph Decomposition

- Example of a 2-dimensional ($K = 2$) decomposition:
Example of a 2-dimensional ($K = 2$) decomposition:
“Multi-dimensional” Filterbanks on graphs

Two-dimensional two-channel filterbank on graphs:

- **Advantages:**
  - Perfect reconstruction and orthogonal for *any* graph and *any* bpt decomposition.
  - Defined metrics to find "good" bipartite decompositions.
Example

(a) Minnesota traffic graph and graph signal

(b)
Bipartite decomposition
Output coefficients of the proposed filterbanks with parameter $m = 24$. 
Reconstructed graph-signals for each channel.
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Depth Image Coding [Narang, Chao and Ortega, 2013]

- Edge Detection
- Graph Selection
- Edge Encoding (JBIG)
- Graph-based Wavelet Transform
- Wavelet Coefficients Encoding (SPIHT)
- Output Bit stream

Advantage:
Link-weights can be adjusted to reflect geometrical structure of the image.
Depth Image Coding [Narang, Chao and Ortega, 2013]

Advantage: Link-weights can be adjusted to reflect geometrical structure of the image.
Depth Image Coding [Narang, Chao and Ortega, 2013]

- Edge detection: Prewitt
- Laplacian Normalization: Random Walk Laplacian
- Filterbanks: GraphBior 4/3 and CDF 9/7
- Unreliable Link Weight: 0.01
- Transform level: 5
- Encoder: SPIHT
What makes these “graph transforms”? 

- **Graph-based shift invariance:**

\[
H = \sum_{k=0}^{L-1} \alpha_k L^k \quad \text{or} \quad H = \sum_{k=0}^{L-1} \alpha_k A^k
\]

- **Graph Fourier Transform**

\[
H = h(\mathcal{L}) = U h(\Lambda) U
\]
Conclusions

- Extending signal processing methods to arbitrary graphs: Downsampling, Space-frequency, Multiresolution, Wavelets
- Many open questions: very diverse types of graphs, results may apply to special classes only

Outcomes
- Work with massive graph-datasets: potential benefits of localized “frequency” analysis
- Novel insights about traditional applications (image/video processing)

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