Graph Signal Processing and Applications [1]

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Nov. 12, 2014

Acknowledgements

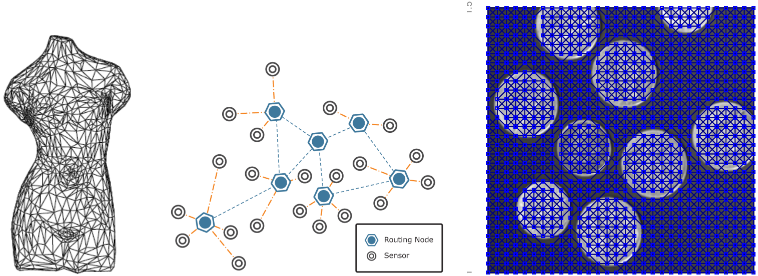
- Collaborators
  - **Dr. Sunil Narang** (Microsoft), **Dr. Godwin Shen** (Northrop-Grumman), **Dr. Eduardo Martínez Enríquez** (Univ. Carlos III, Madrid)
  - **Akshay Gadde, Jessie Chao, Aamir Anis, Yongzhe Wang, Eduardo Pávez, Hilmi Egilmez** (USC)
  - Prof. Marco Levorato (UCI), Prof. Urbashi Mitra (USC) Prof. Fernando Díaz de María (Univ. Carlos III, Madrid), Prof. Gene Cheung (NII), Prof. Pierre Vandergheynst (EPFL), Prof. Pascal Frossard (EPFL), Dr. David Shuman (EPFL).
- Funding
  - Grants: NASA AIST-05-0081, NSF CCF-1018977, CCF-1410009
  - Gifts: MERL, LGE
  - Sabbatical: **Japan Society for Promotion of Science (JSPS)**

Next Section

Motivation

Graphs provide a flexible model to represent many datasets:

- Examples in Euclidean domains

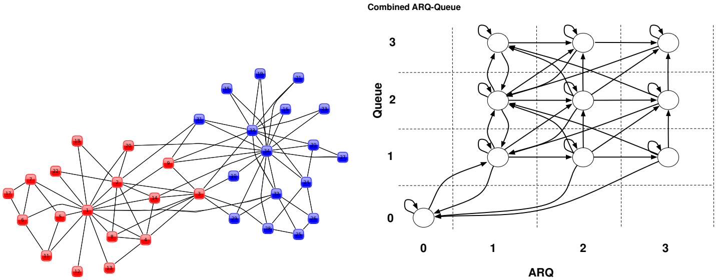


(a) (b) (c)

(a) Computer graphics[2] (b) Wireless sensor networks [3] (c) image - graphs

Motivation

- Examples in non-Euclidean settings



(a) (b)

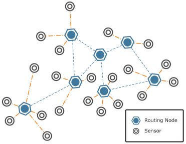
(a) Social Networks [4], (b) Finite State Machines(FSM)

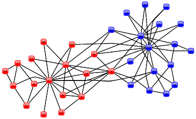
Graphs can capture complex relational characteristics (e.g., spatial, topological).

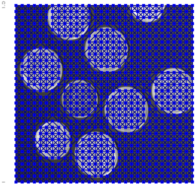
Graph Signal Processing?

- Graph: Assume fixed
- Signal: set of scalars associated to graph vertices
- Define familiar notions: frequency, sampling, transforms, etc
- Use these for compression, denoising, interpolation, etc

Examples

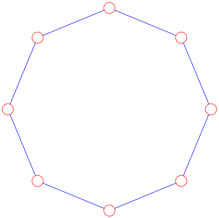






- Sensor network
  - Relative positions of sensors (kNN), temperature
  - does temperature vary smoothly?
- Social network
  - friendship relationship, age
  - are friends of similar age?
- Images
  - pixel positions and similarity, pixel values
  - discontinuities and smoothness

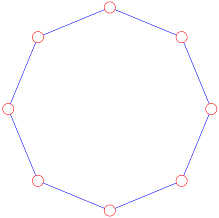
What do we know about transformations on Graphs?

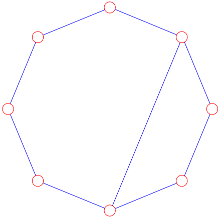


$$\mathbf{L} = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

- $\mathbf{A}$ and $\mathbf{D}$: adjacency and degree matrices, $\mathbf{L} = \mathbf{D} - \mathbf{A}$: graph Laplacian
- $\mathbf{L}$ can be interpreted as a local (high-pass) operation on this graph
- Circulant matrix – Eigenvectors: DFT

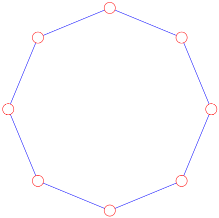
Graphs

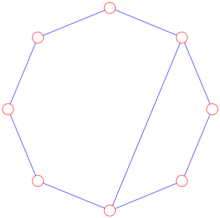




- Can we do similar things on more complex graphs?

Graphs

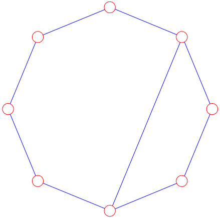




- Can we do similar things on more complex graphs?
- Yes! But things get more complicated

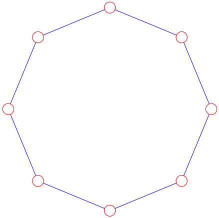
Graphs

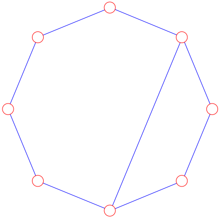
$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & \color{red}{1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & \color{red}{1} & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$



- $\mathbf{A}$ is no longer circulant – no DFT in general, but...
- Polynomials of $\mathbf{L} = \mathbf{D} - \mathbf{A}$ or $\mathbf{A}$ are local operators
- There will be a frequency interpretation: eigenvectors of $\mathbf{L}$.

What makes these “graph transforms”?





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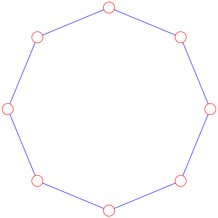

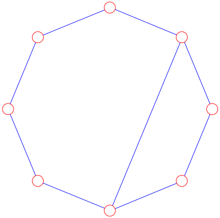

- Graph-based shift invariance – Operator is the same, local variations captured by **A** or **L**.

$$\mathbf{H} = \mathbf{L} = \mathbf{D} - \mathbf{A}$$

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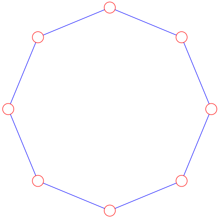
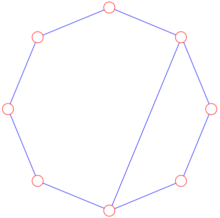




- Graph-based shift invariance – Operator is the same, local variations captured by **A** or **L**.

$$\mathbf{H} = \mathbf{L} = \mathbf{D} - \mathbf{A}$$

- This can be generalized:

$$\mathbf{H} = \sum_{k=0}^{L-1} \alpha_k \mathbf{L}^k \quad \text{or} \quad \mathbf{H} = \sum_{k=0}^{L-1} \alpha_k \mathbf{A}^k$$

- Or alternatively, based on Graph Fourier Transform

Summary

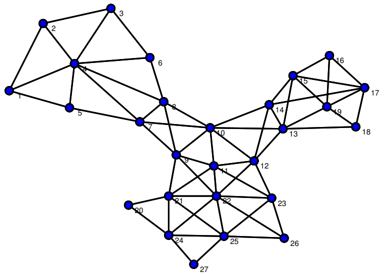
- Localized linear operations on graphs using polynomials of **A** or **L**.
- Frequency interpretation is possible for eigenvectors of **A** or **L**.
- A great deal depends on the topology of the graph

- In what follows we consider mostly undirected graphs without self loops and use **L**.
  [Shuman, Narang, Frossard, Ortega, Vandergheysnt, SPM'2013]
- Other approaches are possible based on **A**
  [Sandryhaila and Moura 2013]

Research Goals

- Extend signal processing methods to arbitrary graphs
  - Downsampling, graph-frequency localization, multiresolution, wavelets, interpolation
- Outcomes
  - Work with massive graph-datasets: localized "frequency" analysis
  - Novel insights about traditional applications (image/video processing)
  - New applications
- This talk
  - Graph Signal Processing – intro
  - Graph Filterbank design
  - Applications
    - Depth image coding
    - Semi-supervised learning (2nd talk!)

Graphs 101



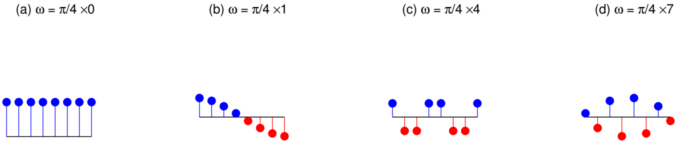
- Graph $G = (\mathcal{V}, E, w)$.
- Adjacency matrix $\mathbf{A}$
- Degree matrix $\mathbf{D} = diag\{d_i\}$
- Laplacian matrix $\mathbf{L} = \mathbf{D} - \mathbf{A}$.
- Normalized Laplacian matrix $\mathcal{L} = \mathbf{D}^{-1/2}\mathbf{L}\mathbf{D}^{-1/2}$
- Graph Signal $\mathbf{f} = \{f(1), f(2), ..., f(N)\}$

- Assumptions:
  1. Undirected graphs without self loops.
  2. Scalar sample values

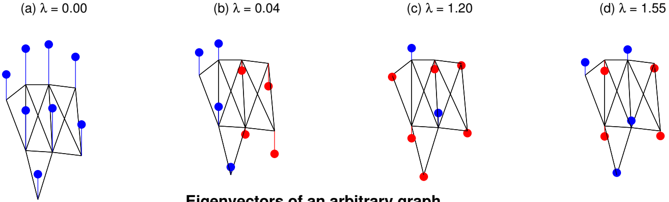
Spectrum of Graphs

- Graph Laplacian Matrix $\mathbf{L} = \mathbf{D} - \mathbf{A} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}'$
- Eigen-vectors of $\mathbf{L}$: $\mathbf{U} = \{\mathbf{u}_k\}_{k=1:N}$
- Eigen-values of $\mathbf{L}$: $diag\{\mathbf{\Lambda}\} = \lambda_1 \leq \lambda_2 \leq ... \leq \lambda_N$
- **Eigen-pair system $\{(\lambda_k, \mathbf{u}_k)\}$ provides Fourier-like interpretation — Graph Fourier Transform (GFT)**

Graph Frequencies

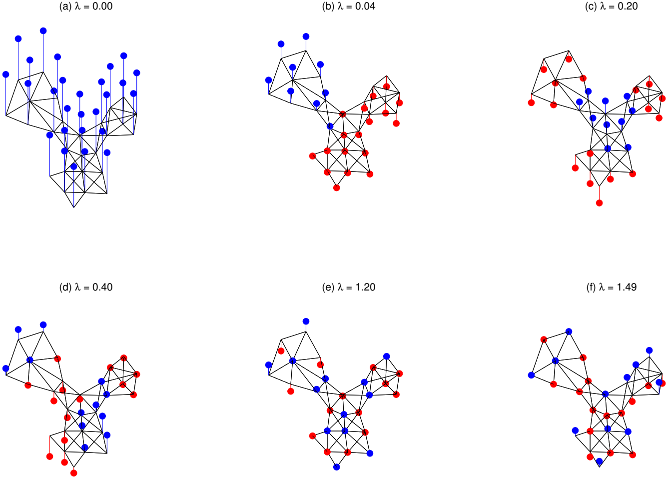


DCT basis for regular signals

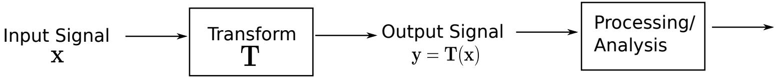


Eigenvectors of an arbitrary graph

Eigenvectors of graph Laplacian



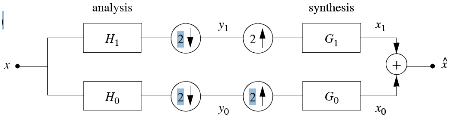
Graph Transforms



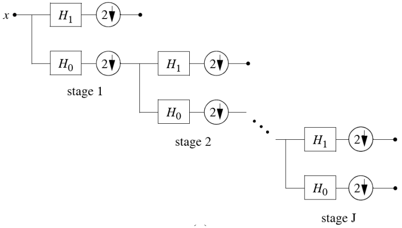
- Desirable properties
  - Invertible
  - Critically sampled
  - Orthogonal
  - Localized in graph (space) and graph spectrum (frequency)
- Local Linear Transform
- Can we define Graph Wavelets?

Next Section

Discrete Wavelet Transforms in 2 slides – 1



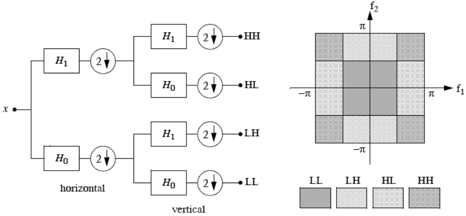
(d)



(e)

From Vetterli and Kovacevic, Wavelets and Subband Coding, '95

Discrete Wavelet Transforms in 2 slides – 2



(a)

(b)

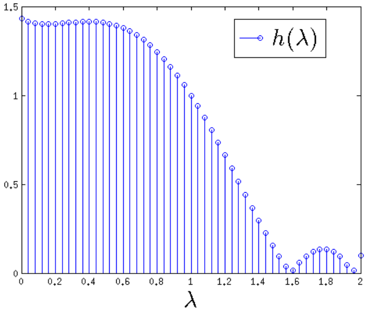
Note: Filters have some frequency and space localization

From Vetterli and Kovacevic, [Ding'07]

Prior Work – Spectral Graph Transforms

- Designed in the spectral domain of the graph. Examples:
  - Diffusion Wavelets [Coifman and Maggioni 2006]
  - Spectral Wavelets on Graphs [Hammond et al. 2011]

- Spectral Wavelet transforms [Hammond et al. 2011]:
  Design spectral kernels: $h(\lambda) : \sigma(G) \to \mathbb{R}$.



$$\mathbf{T}_h = h(\mathcal{L}) = \mathbf{U}h(\mathbf{\Lambda})\mathbf{U}^t$$

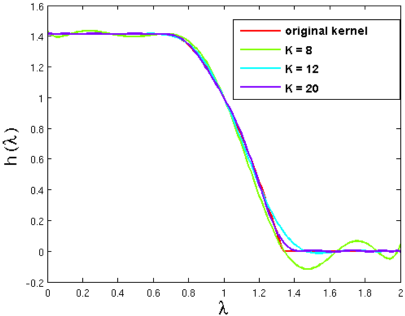
where
$h(\mathbf{\Lambda}) = diag\{h(\lambda_i)\}$

Spectral Graph Transforms Cont'd

- Output Coefficients:

$$\mathbf{w}_f = \mathbf{T}_h\mathbf{f} = \sum_{\lambda\in\sigma(G)} h(\lambda).\bar{f}(\lambda)\mathbf{u}_\lambda$$

- Polynomial kernel approximation:



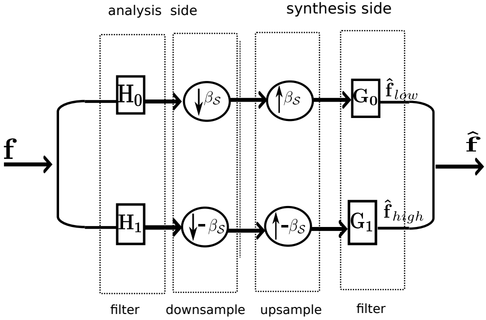
$$h(\lambda) \approx \sum_{k=0}^{K} a_k\lambda^k$$

$$\mathbf{T}_h \approx \sum_{k=0}^{K} a_k\mathcal{L}^k$$

K-hop localized: no spectral decomposition required.

Graph Filterbank Designs

- Formulation of critically sampled graph filterbank design problem
- Design filters using spectral techniques [Hammond et al. 2009].
- Orthogonal (not compactly supported) [IEEE TSP June 2012]
- Bi-Orthogonal (compactly supported) [IEEE TSP Oct 2013]

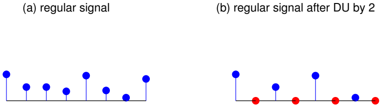


Downsampling/Upsampling in Graphs

Downsampling-upsampling operation:

- Regular Signals:

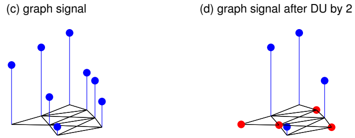
$$f_{du}(n) = \begin{cases} f(n) & \text{if } n = 2m \\ 0 & \text{if } n = 2m+1 \end{cases}$$



- Graph signals:

$$f_{du}(n) = \begin{cases} f(n) & \text{if } n \in \mathcal{S} \\ 0 & \text{if } n \notin \mathcal{S} \end{cases}$$

for some set $\mathcal{S}$.



- For regular signals DU by 2 operation is equivalent to $F_{du}(e^{j\omega}) = 1/2(F(e^{j\omega}) + F(e^{-j\omega}))$ in the DFT domain.
- What is the DU by 2 for graph signals in GFT domain?

Downsampling in Graphs

- Define $\mathbf{J}_\beta = \mathbf{J}_{\beta_H} = diag\{\beta_H(n)\}$.
- In vector form:

$$
\begin{aligned}
\mathbf{f}_{du} &= \frac{1}{2}(\mathbf{f} + \mathbf{J}_\beta \mathbf{f}) \\
&= \frac{1}{2}(\mathbf{f} + \tilde{\mathbf{f}})
\end{aligned}
$$

Downsampling in Graphs

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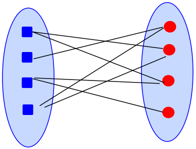
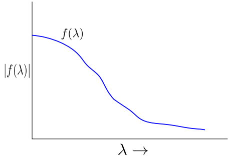
- Spectral Folding [4]: For a bipartite graph $\tilde{f}(\lambda) = f(2 - \lambda)$.





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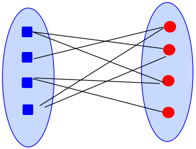
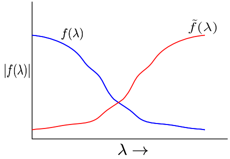
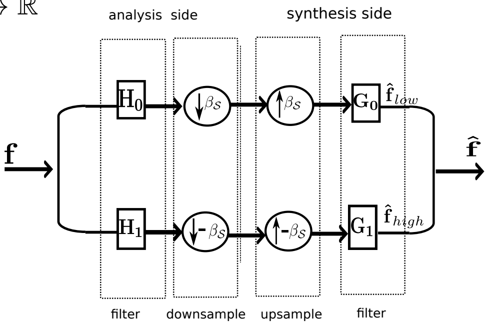




Graph filterbanks

- Filters designed in spectral domain (as [Hammond et al, 2009])
- Analysis:
  - $h_i(\lambda) : \mathbb{R} \to \mathbb{R}$ for $i = 0, 1$
  - $\mathbf{H}_i = h_i(\mathcal{L}) = \mathbf{U} h_i(\mathbf{\Lambda}) \mathbf{U}^t$
- Synthesis:
  - $g_i(\lambda) : \mathbb{R} \to \mathbb{R}$
  - $\mathbf{G}_i = g_i(\mathcal{L})$



Wavelet filterbanks on bipartite graphs

- Aliasing Cancellation $\Rightarrow$ $\mathbf{B} = 0$ if for all $\lambda \in \sigma(G)$:

$$B(\lambda) = g_1(\lambda)h_1(2-\lambda) - g_0(\lambda)h_0(2-\lambda) = 0$$

Wavelet filterbanks on bipartite graphs

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- Perfect Reconstruction $\Rightarrow \mathbf{A} = c\mathbf{I}$ if for all $\lambda \in \sigma(G)$:

$$A(\lambda) = g_1(\lambda)h_1(\lambda) + g_0(\lambda)h_0(\lambda) = c$$

GraphBior design

- Analogous to CDF wavelet Filters [Narang and Ortega, IEEE TSP, 2013]

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- Choose kernels, s.t.,

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- The PR condition ($\mathbf{A} = 0$) becomes:

$$\underbrace{h_1(\lambda)g_1(\lambda)}_{p(\lambda)} + \underbrace{h_1(2-\lambda)g_1(2-\lambda)}_{p(2-\lambda)} = c$$

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- Design $p(\lambda)$ as a "maximally flat" polynomial and factorize into $h_1(\lambda)$, $g_1(\lambda)$ terms. Exact reconstruction with polynomial filter (compact support).

Bipartite Subgraph Decomposition

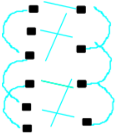
- But not all graphs are bipartite...

Bipartite Subgraph Decomposition

- But not all graphs are bipartite...
- Solution: "Iteratively" decompose non-bipartite graph G into K bipartite subgraphs:
  - each subgraph covers the same vertex set.
  - each edge in G belongs to exactly one bipartite graph.

Bipartite Subgraph Decomposition

- Example of a 2-dimensional ($K = 2$) decomposition:



G_0

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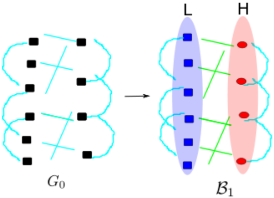

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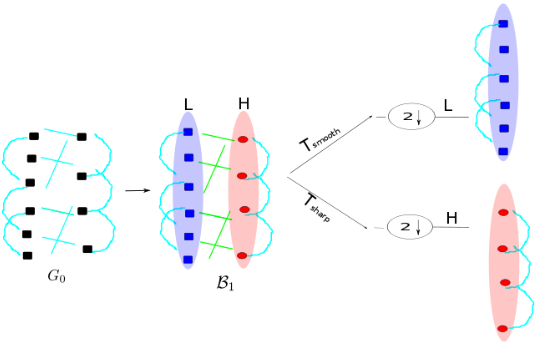
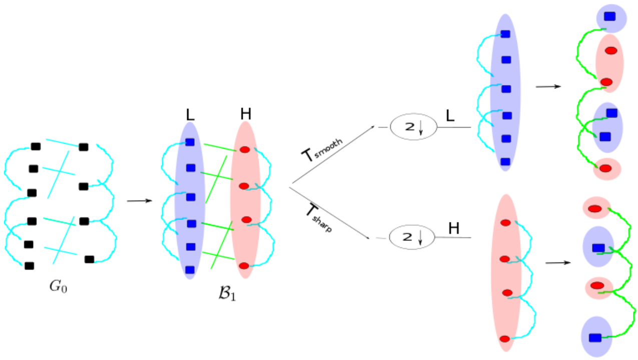
- Example of a 2-dimensional ($K = 2$) decomposition:



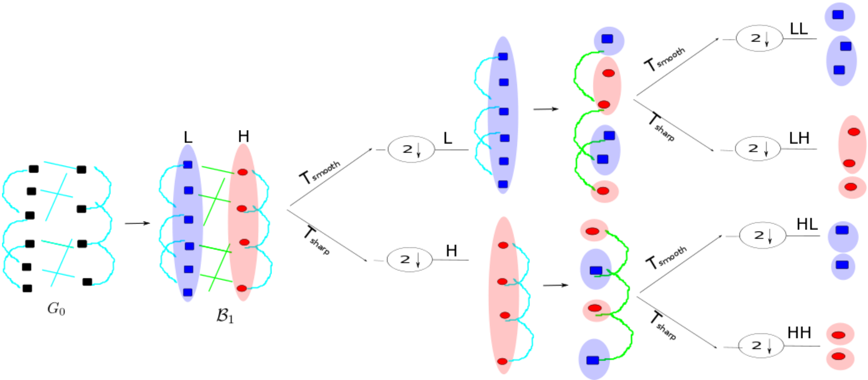
Bipartite Subgraph Decomposition

- Example of a 2-dimensional ($K = 2$) decomposition:



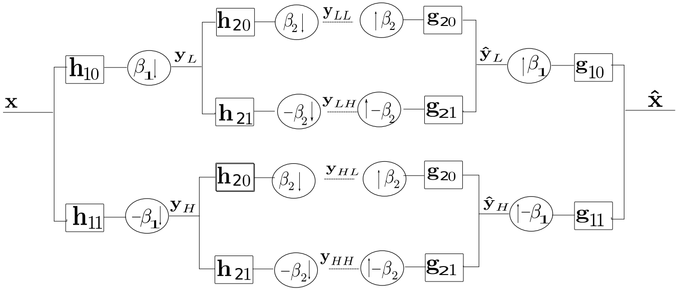
Bipartite Subgraph Decomposition

- Example of a 2-dimensional ($K = 2$) decomposition:



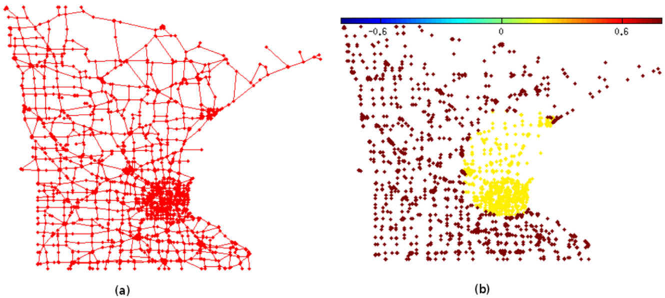
"Multi-dimensional" Filterbanks on graphs

Two-dimensional two-channel filterbank on graphs:



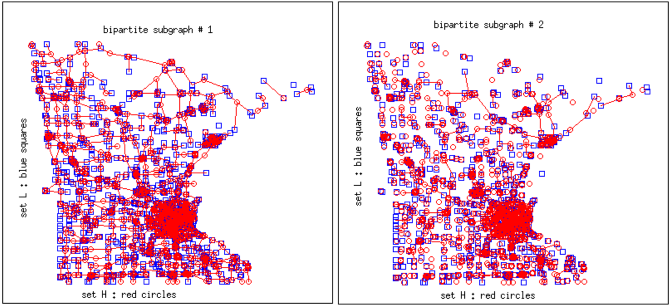
- **Advantages:**
  - Perfect reconstruction and orthogonal for *any* graph and *any* bpt decomposition.
  - defined metrics to find "good" bipartite decompositions.

Example



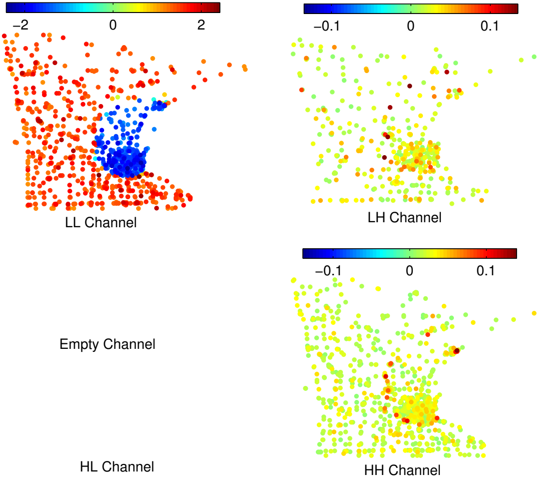
Minnesota traffic graph and graph signal

Example



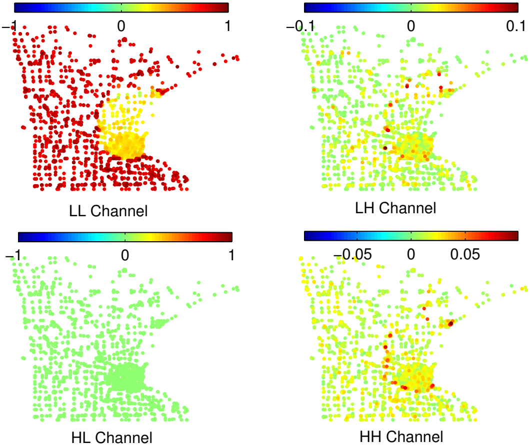
Bipartite decomposition

Example



Output coefficients of the proposed filterbanks with parameter $m = 24$.

Example

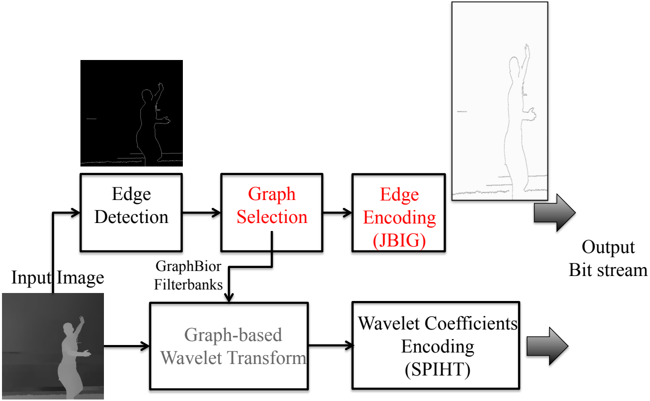


Reconstructed graph-signals for each channel.

Next Section

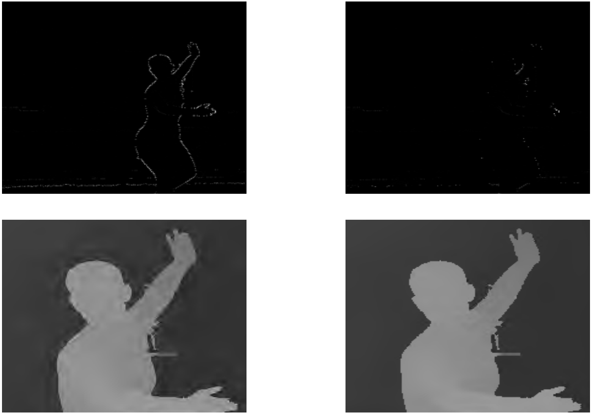
Depth Image Coding [Narang, Chao and Ortega, 2013]

- Block Diagram



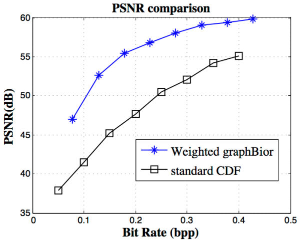
Depth Image Coding [Narang, Chao and Ortega, 2013]

CDF 9/7 | Graph 9/7



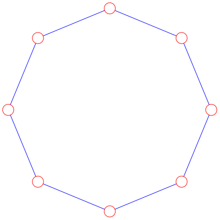
Depth Image Coding [Narang, Chao and Ortega, 2013]

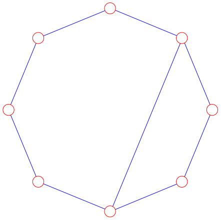
- Edge detection: Prewitt
- Laplacian Normalization: Random Walk Laplacian
- Filterbanks: GraphBior 4/3 and CDF 9/7
- Unreliable Link Weight: 0.01
- Transform level: 5
- Encoder: SPIHT



Next Section

What makes these "graph transforms"?





- Graph-based shift invariance:

$$\mathbf{H} = \sum_{k=0}^{L-1} \alpha_k \mathbf{L}^k \quad \text{or} \quad \mathbf{H} = \sum_{k=0}^{L-1} \alpha_k \mathbf{A}^k$$

- Graph Fourier Transform

$$\mathbf{H} = h(\mathcal{L}) = \mathbf{U} h(\mathbf{\Lambda}) \mathbf{U}$$

Conclusions

- Extending signal processing methods to arbitrary graphs: Downsampling, Space-frequency, Multiresolution, Wavelets
- Many open questions: very diverse types of graphs, results may apply to special classes only
- Outcomes
  - Work with massive graph-datasets: potential benefits of localized "frequency" analysis
  - Novel insights about traditional applications (image/video processing)
- To get started: [Shuman, Narang, Frossard, Ortega, Vandergheysnt, SPM'2013]

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