Signal Processing on Graphs: Recent Results, Challenges and Applications

Oversampled Graph Transforms and Image Processing

Yuichi Tanaka

Tokyo University of Agriculture and Technology

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MSP Lab
Multidimensional media Signal Processing
Graduate School of BASE, TUAT

Collaborators & Acknowledgments

- Collaborators
  - S. Ono & M. Yamagishi (Tokyo Institute of Technology)
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Outline

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Conventional Graph Filter Banks (GFBs)

- **Undecimated GFBs** (redundancy: M)
  - Spectral graph wavelets (SGWT) [Hammond et al. 2011]
  - Tight graph wavelets [Leonardi et al. 2013], [Shuman et al. 2014]
- **Critically sampled GFBs** (redundancy: 1)
  - Two-channel orthogonal GFBs [Narang and Ortega 2012]
  - Two-channel biorthogonal GFBs [Narang and Ortega 2013]

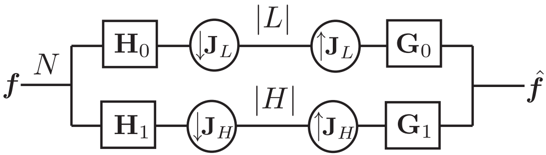


Figure: Critically sampled graph filter bank.

Oversampled Graph Transforms

- Graph transforms with redundancy R $(1 < R < M)$
- Topics:
  - Perfect reconstruction condition
  - Redundancy
  - Design method

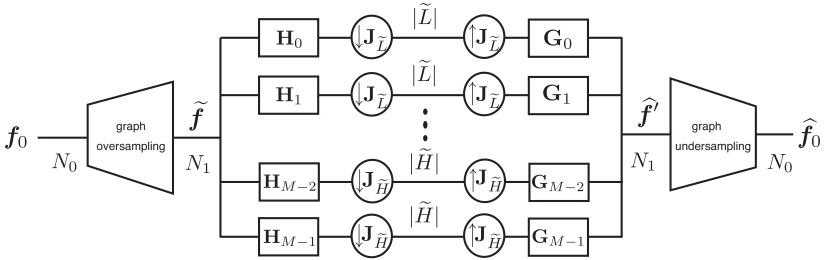


Figure: Graph oversampling followed by M-channel oversampled graph filter bank.

Oversampling Graph Signals

Two possibilities to oversample graph signals:

1. Oversampling-then-filtering by **oversampled graph Laplacian matrix** [Sakiyama and Tanaka 2014]
   - Simultaneous oversampling of graph signal and graph Laplacian matrix
2. Transformation by M (> 2) filters by **oversampled GFBs** [Tanaka and Sakiyama 2014]

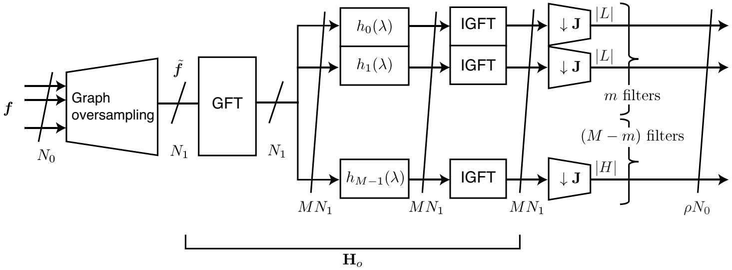


Figure: Signal processing flow of oversampled graph filter bank. Symbols just below the skewed lines indicate the number of signals at typical positions.

Outline

Oversampled Graph Laplacian Matrix

- Original GLM of size $N_0 \times N_0$: $\mathbf{L}_0 = \mathbf{D}_0 - \mathbf{A}_0$
- **Oversampled GLM** of size $N_1 \times N_1$

$$\widetilde{\mathcal{L}} = \widetilde{\mathbf{D}}^{-1/2}\widetilde{\mathbf{L}}\widetilde{\mathbf{D}}^{-1/2}$$

where

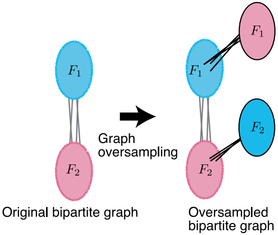
$$\widetilde{\mathbf{L}} = \widetilde{\mathbf{D}} - \widetilde{\mathbf{A}}$$

$$\widetilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A}_0 & \mathbf{A}_{01} \\ \mathbf{A}_{01}^T & \mathbf{0}_{N_1-N_0} \end{bmatrix}$$

  - $\widetilde{\mathbf{A}}$, $\widetilde{\mathbf{D}}$: oversampled adjacency/degree matrix
  - $\mathbf{A}_{01}$: edge information between original nodes and appended ones

Effective Graph Expansion Methods

- How to choose a good $\mathbf{A}_{01}$?
  - **Oversampled bipartite graphs with all edges of the original graph**
- If oversampled graph is bipartite:
  - GFBs with downsampling can be used.
  - We can control the overall redundancy.
- Our work (partially) answers the following questions:
  - Can *any* graphs be converted into an oversampled bipartite graph?
  - Systematic construction method?
  - Relationship to graph theory?



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Conversion of K-colorable graph

- One K-colorable graph $\rightarrow \lceil \log_2 K \rceil$ bipartite subgraphs [Harary et al. 1977], [Narang and Ortega 2012]
- **Bipartite subgraphs can be merged into one OS bipartite graph.**
- Step-by-step example: Conversion of 3-colorable graph

Definition

- F_1 to F_3: Colored node sets
- $\mathcal{B}_1$: Subgraph 1 (having edges linking $F_1 \cup F_2$ and F_3)
- $\mathcal{B}_2$: Subgraph 2 (having edges linking F_1 and F_2)

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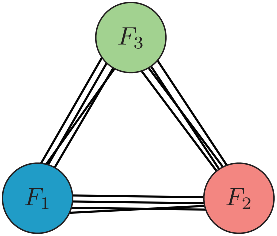
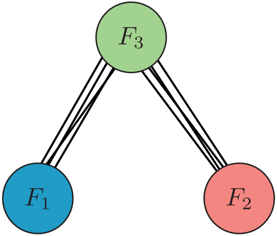
Oversampled Graph Construction

Step 1: Decide **foundation bipartite graph**

- Foundation bipartite graph: 1st (or ground) floor of OS graph
- In this example, we chose $\mathcal{B}_1$ as the foundation bipartite graph.



(a) Original graph



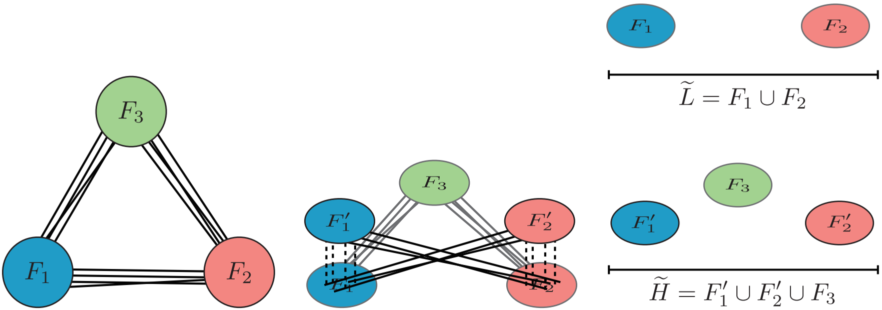
(b) Foundation bpt. graph

Figure: OS bpt. graph construction

Oversampled Graph Construction

Step 2: Append and connect **oversampled nodes**

1. Additional nodes F_1' and F_2' are placed just above F_1 and F_2.
2. Connect F_1' and F_2 and F_2' and F_1.

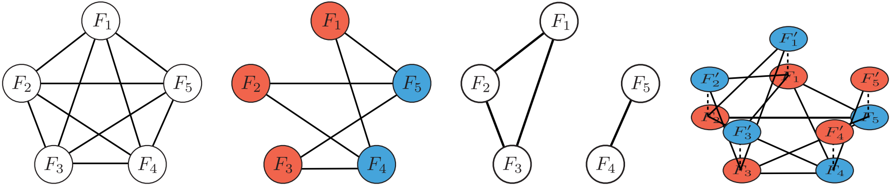


(a) Original graph (b) OS bpt. graph (c) $\widetilde{L}$ and $\widetilde{H}$

Figure: OS bpt. graph construction

K-Colorable Case

- Similar construction to the 3-colorable case
- Freedom for choosing the foundation bipartite graph

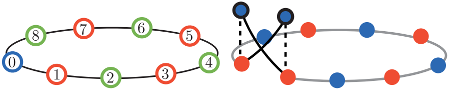


(a) Original graph (b) Foundation bpt. graph (c) Remaining edges (d) OS bpt. graph

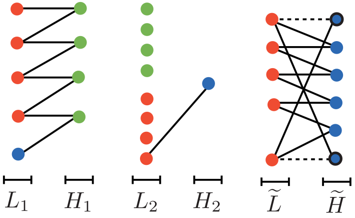
Figure: OS bpt. graph construction for a five-colorable graph

Example: Graph Oversampling of Ring Graph

- Ring graph with odd # of nodes: 3-colorable
- Critically sampled bpt. graphs
  - # of nodes are heavily biased.
  - A few relationships btw nodes becomes very weak.
- Oversampled bpt. graphs
  - # of nodes are (almost) even.
  - Small redundancy: $(2n+3)/(2n+1)$
  - Strong connection



(a) Original graph (b) OS bpt. graph

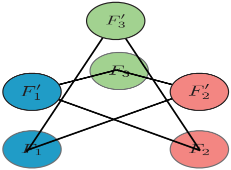


(c) Bpt. subgraph 1 (d) Bpt. subgraph 2 (e) $\widetilde{L}$ and $\widetilde{H}$

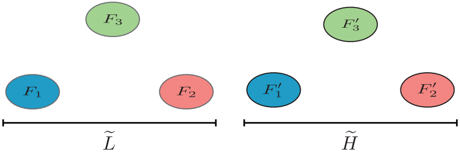
Relationship w/ Bipartite Double Cover (BDC)

BDC of $\mathcal{G}$: $\widetilde{\mathcal{G}}_{BDC} = \mathcal{G} \otimes K_2$ (K_2: complete graph of two vertices)

- Adjacency matrix: $\widetilde{\mathbf{A}}_{BDC} = \begin{bmatrix} \mathbf{0} & \mathbf{A} \\ \mathbf{A} & \mathbf{0} \end{bmatrix}$
- Normalized GLM: $\widetilde{\mathcal{L}}_{BDC} = \begin{bmatrix} \mathbf{I} & -\mathbf{D}^{-\frac{1}{2}}\mathbf{A}\mathbf{D}^{-\frac{1}{2}} \\ -\mathbf{D}^{-\frac{1}{2}}\mathbf{A}\mathbf{D}^{-\frac{1}{2}} & \mathbf{I} \end{bmatrix}$
- $2N$ nodes and $2|\mathcal{E}|$ edges in $\widetilde{\mathcal{G}}_{BDC}$ (redundancy: 2)



(f) BDC



(g) $\widetilde{L}$ and $\widetilde{H}$

Figure: Bipartite graph cover for a 3-colorable graph

Graph Fourier Spectrum of BDC

- Eigenvalues and eigenvectors
  - $\boldsymbol{u}_{\lambda_i}$: eigenvector of $\mathcal{L}$ (original graph)

Eigenvectors of $\widetilde{\mathcal{L}}_{BDC}$

$$\widetilde{\boldsymbol{u}}_{\widetilde{\lambda}_i} = \frac{1}{\sqrt{2}}[\boldsymbol{u}_{\lambda_i}^T \ \boldsymbol{u}_{\lambda_i}^T]^T \ (\widetilde{\lambda}_i = \lambda_i), \ \frac{1}{\sqrt{2}}[\boldsymbol{u}_{\lambda_i}^T \ -\boldsymbol{u}_{\lambda_i}^T]^T \ (\widetilde{\lambda}_i = 2 - \lambda_i)$$

- Graph Fourier coefficients of $\widetilde{\boldsymbol{f}} = [\boldsymbol{f}_0^T \ \boldsymbol{f}_0^T]^T$
  - $\widetilde{\boldsymbol{u}}_{\lambda_i}^T \widetilde{\boldsymbol{f}} = \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{u}_{\lambda_i}^T & \boldsymbol{u}_{\lambda_i}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_0 \\ \boldsymbol{f}_0 \end{bmatrix} = \sqrt{2}\boldsymbol{u}_{\lambda_i}^T \boldsymbol{f}_0 = \sqrt{2}\hat{f}_0(\lambda_i)$
  - $\widetilde{\boldsymbol{u}}_{2-\lambda_i}^T \widetilde{\boldsymbol{f}} = \frac{1}{\sqrt{2}} \begin{bmatrix} \boldsymbol{u}_{\lambda_i}^T & -\boldsymbol{u}_{\lambda_i}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_0 \\ \boldsymbol{f}_0 \end{bmatrix} = 0$
  - # of nonzero coefficients: N
- **Spectrum of BDC = Spectrum of the original graph**

Graph Fourier Spectrum of OS Bipartite Graph

- Adjacency and degree matrix: $\widetilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A}_f & \mathbf{A}_r \\ \mathbf{A}_r & \mathbf{0} \end{bmatrix}$, $\widetilde{\mathbf{D}} = \begin{bmatrix} \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_r \end{bmatrix}$
  - $\mathbf{A}_f$: Adjacency matrix of the foundation bpt. graph
  - $\mathbf{A}_r$: Adjacency matrix of the remaining graph
  - $\mathbf{A} = \mathbf{A}_f + \mathbf{A}_r$, $\mathbf{D} = \mathbf{D}_f + \mathbf{D}_r$
- Normalized GLM: $\widetilde{\mathcal{L}} = \begin{bmatrix} \mathbf{I} - \mathbf{D}^{-\frac{1}{2}}\mathbf{A}_f\mathbf{D}^{-\frac{1}{2}} & -\mathbf{D}^{-\frac{1}{2}}\mathbf{A}_r\mathbf{D}_r^{-\frac{1}{2}} \\ -\mathbf{D}_r^{-\frac{1}{2}}\mathbf{A}_r\mathbf{D}^{-\frac{1}{2}} & \mathbf{I} \end{bmatrix}$
- $\widetilde{\mathcal{L}}$ has the eigenvector $\widetilde{\boldsymbol{u}}_{\lambda_i} = \frac{1}{\sqrt{2}}[\boldsymbol{u}_{\lambda_i}^T \ \boldsymbol{u}_{\lambda_i}^T]^T$ with $\widetilde{\lambda}_i = \lambda_i \iff \mathbf{D}_r = \mathbf{D}$ (= BDC)
- $\rightarrow$ **BDC is a special case of the oversampled GLM.**

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Experimental Results

Nonlinear approximation of images

- All lowpass + some fractions of highpass coefficients
- Compared with
  - CDF 9/7 DWT
  - Laplacian pyramid with 9/7 DWT
  - GraphBior [Narang and Ortega 2012]
  - Graph Laplacian pyramid [Shuman et al. 2013]

Figure: Left: *Cameraman* (256 × 256). Right: *Ballet* (512 × 512).

Experimental Results

NLA: PSNR comparison

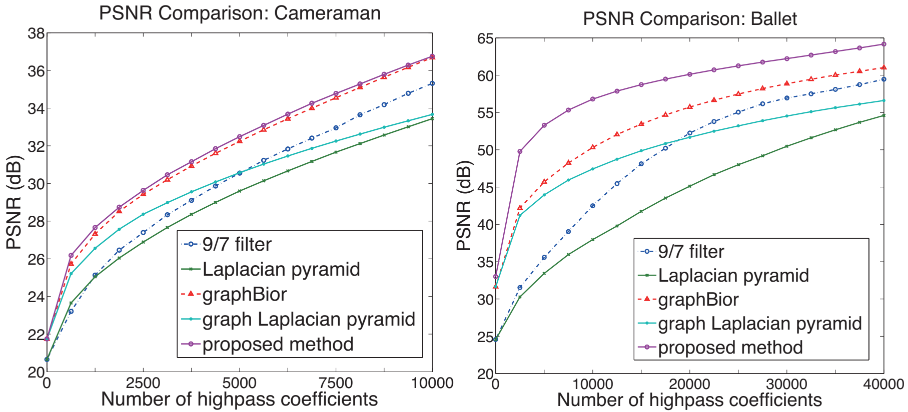


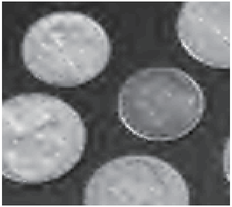
Figure: Left: *Cameraman*. Right: *Ballet*.

Experimental Results

Side-by-side quality comparison: All lowpass + 3% highpass

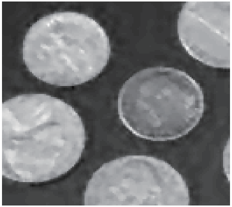
(a) Original

(b) 9/7 DWT



(c) LP

(d) GraphBior



(e) Graph LP

(f) OSGLM

Figure: Reconstructed *Coins* image

Experimental Results

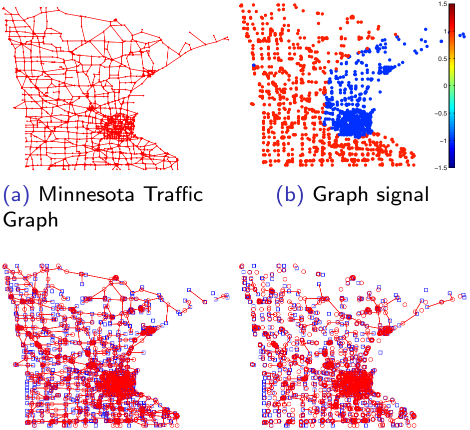
Graph signal denoising: SNR comparison

Table: Denoised Results of Minnesota Traffic Graph: SNR (dB)

σ	noisy	sym8 (1 level)	sym8 (5 levels)	graphBior (CSGLM)	GLP
1/32	30.15	30.17	30.22	31.44	31.39
1/16	24.08	24.25	24.07	25.61	25.68
1/8	18.06	18.65	17.99	19.97	20.02
1/4	12.02	11.94	11.07	14.19	14.24
1/2	5.99	6.23	5.76	8.50	8.51
1	-0.02	1.59	3.13	2.63	2.61
Redundancy	-	1.00	1.00	1.00	2.05
σ	SGWT	OSGFB (CSGLM)	graphBior (BDC)	graphBior (OSGLM)	OSGFB (OSGLM)
1/32	33.35	34.75	32.54	32.46	**35.08**
1/16	27.76	28.78	26.75	26.76	**29.34**
1/8	22.08	21.84	20.81	20.88	**23.17**
1/4	15.05	15.26	14.79	14.94	**17.63**
1/2	10.33	10.29	8.92	9.00	**12.31**
1	**8.82**	4.24	3.03	3.11	7.04
Redundancy	4.00	4.00	2.00	1.37	2.74

Experimental Results

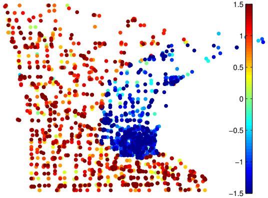
Graph signal denoising: Side-by-side comparison



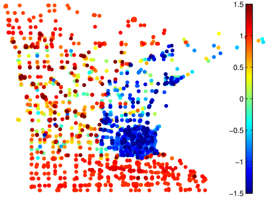
(a) Minnesota Traffic Graph (b) Graph signal

(c) Bpt. subgraph 1 (d) Bpt. subgraph 2

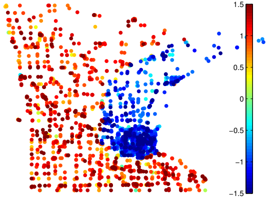
Figure: *Minnesota Traffic Graph*



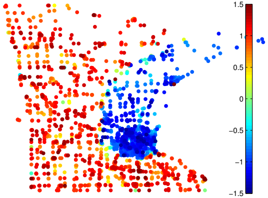
(a) Noisy observation (5.99 dB)



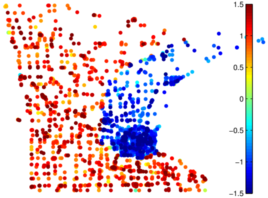
(b) sym8 (5 level): $R = 1.00$ (5.76 dB)



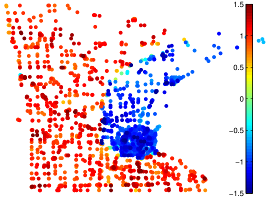
(c) graphBior: $R = 1.00$ (8.50 dB)



(d) SGWT: $R = 4.00$ (10.33 dB)



(e) graphBior with OSGLM: $R = 1.37$ (9.00 dB)



(f) OSGFB with OSGLM: $R = 2.74$ (12.31 dB)

Figure: Denoising results of *Minnesota Traffic Graph*.

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Notations

- Eigenspace projection matrix: $\widetilde{\mathbf{P}}_{\lambda_i} := \sum_{\lambda=\lambda_i} \widetilde{\boldsymbol{u}}_\lambda \widetilde{\boldsymbol{u}}_\lambda^T$
- k-th analysis filter: $\mathbf{H}_k = \sum_{\lambda_i \in \sigma(\widetilde{\mathcal{L}})} h_k(\lambda_i)\widetilde{\mathbf{P}}_{\lambda_i}$ (synthesis: $\mathbf{G}_k$)
- $h_k(\lambda)$, $g_k(\lambda)$: filter kernel (real function for $0 \leq \lambda \leq \lambda_{\max} = 2$)
  - $\boldsymbol{f}_{\text{out}} = \mathbf{U}\text{diag}\{h_k(\lambda_i)\}\mathbf{U}^T\boldsymbol{f}_{\text{in}}$

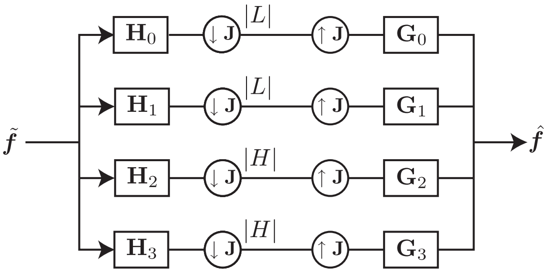

Figure: Oversampled graph filter bank.

Perfect Reconstruction Condition

Transfer function

$$\mathbf{T} = \frac{1}{2}\sum_{\lambda_i}\sum_{k=0}^{M-1}\underbrace{g_k(\lambda_i)h_k(\lambda_i)}_{\text{Amplitude term}}\widetilde{\mathbf{P}}_{\lambda_i} + \frac{1}{2}\sum_{\lambda_i}\sum_{k=0}^{\frac{M}{2}-1}\underbrace{\{-g_k(\lambda_i)h_k(2-\lambda_i)+g_{k+\frac{M}{2}}(\lambda_i)h_{k+\frac{M}{2}}(2-\lambda_i)\}}_{\text{Spectral folding term}}\widetilde{\mathbf{P}}_{\lambda_i}\mathbf{J}$$

leads to the following condition similar to the critically sampled case:

- $\displaystyle\sum_{k=0}^{M-1} g_k(\lambda)h_k(\lambda) = 2$
- $\displaystyle\sum_{k=0}^{\frac{M}{2}-1} -g_k(\lambda_i)h_k(2-\lambda_i)+g_{k+\frac{M}{2}}(\lambda_i)h_{k+\frac{M}{2}}(2-\lambda_i) = 0$

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Perfect Reconstruction Condition

- **Spectral folding effect** can be cancelled with the following filter selection:

$$g_k(\lambda) = h_{k+M/2}(2-\lambda),\ g_{k+M/2}(\lambda) = h_k(2-\lambda)$$

- Product filters $p_k(\lambda) = g_k(\lambda)h_k(\lambda)$ must satisfy

$$\sum_{k=0}^{M/2-1} p_k(\lambda) + p_k(2-\lambda) = 2$$

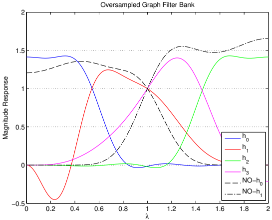
Design of Oversampled Graph Filter Banks

Design methodology

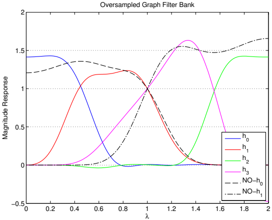
1. Design two-channel halfband filters.
2. Design arbitrary $M-2$ filters.
3. Subtract $M-2$ product filters from the two-channel halfband filter and factorize it to obtain remaining 2 filters.

Design Examples

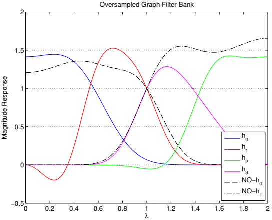
- OSGFB designs for different # of zeros of halfband filters
- Filter length (polynomial order): 10 for lowpass 11 for highpass



(a) $(2, 2)$ zeros



(b) $(4, 4)$ zeros



(c) $(8, 8)$ zeros

Figure: Four-channel oversampled graph filter banks (black lines indicate graphBior(6,6) [Narang and Ortega 2013])

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Graph Signal Decomposition

1. *Coins* image
   - Edge-aware image graph
2. *Minnesota Traffic Graph*
   - Three-colorable graph

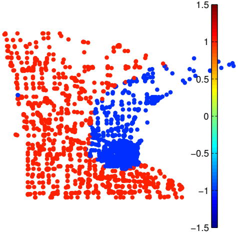

Figure: Graph signals used. Left: *Coins*. Right: *Minnesota Traffic Graph*

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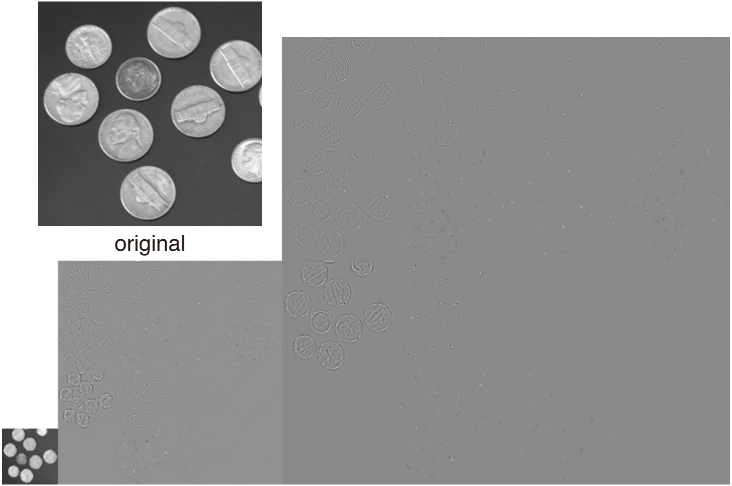

Figure: Multiresolution *Coins* image after three-level decomposition using the oversampled graph filter bank. The original image on the same scale is shown at the top right. The values of the transformed coefficients are scaled to be in the range $[0, 1]$ for the sake of visualization.

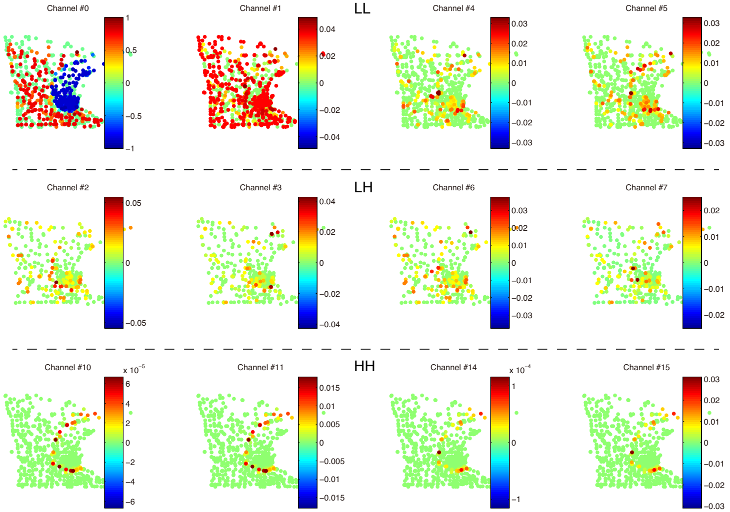

Figure: Graphs decomposed by the proposed oversampled graph filter bank. We use a two-dimensional four-channel filter bank leading to $4^2 = 16$ channels. Note that the graph is three-colorable: therefore, channels 8, 9, 12, and 13 (corresponding to the HL channel for the critically sampled filter banks) are empty.

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BM3D: Image Denoising State-of-the-Art

Two-step algorithm in BM3D [Dabov et al. 2007]

1. Basic estimate
   1.1 Grouping
   1.2 Collaborative separable filtering + hard-thresholding
   1.3 Aggregation
2. Final estimate
   2.1 Grouping
   2.2 Collaborative Wiener filtering
   2.3 Aggregation

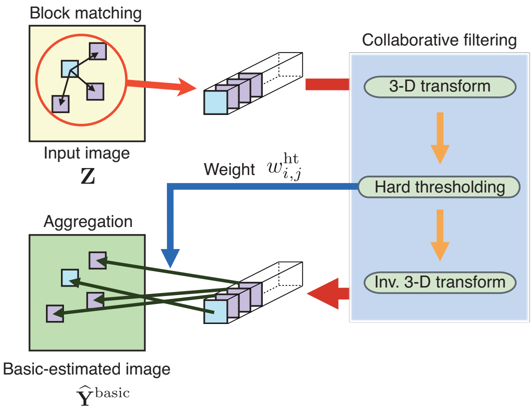
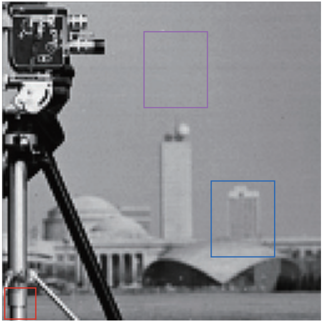


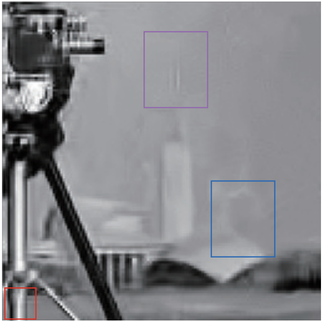
Figure: Basic estimate of BM3D

Problem on BM3D

- BEST **objective performance**
- Problems due to aggregation and Wiener filtering
  1. **Unnatural artifacts**
  2. **Oversmoothing**



(a)



(b)

Figure: (a) original, (b) denoised image by BM3D

Collaborative Graph Wavelet Shrinkage

- **Improved basic estimate step** based on graph signal processing
- Algorithm
  1. Grouping
  2. **Inter-/Intra-patch graph construction**
  3. **Collaborative filtering with graph wavelets** + hard-thresholding
  4. Aggregation

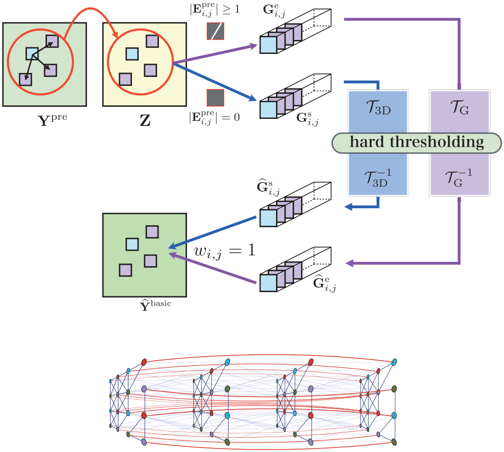


Figure: CGWS

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Experiments on Depth Map Denoising

- Depth map: Piecewise smooth (constant) images
- Depth maps used (all from *Middlebury Stereo Datasets*)
  - *art* (512×512), *ballet* (512×512), *Ryerson* (512×512), *cones* (450×375), *teddy* (450×375)
- Corrupted by AWGN with $\sigma = 10, 30, 50, 100$
- Comparison with
  - Gaussian filtering + L0 Smoothing [Xu et al. 2011]
  - K-SVD [Aharon and Elad 2006]
  - BM3D

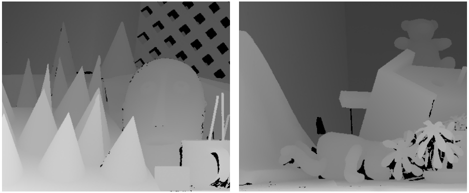

Figure: Left: *cones*. Right: *teddy*.

Experimental Results: PSNR Comparisons

σ/PSNR	method	*art*	*ballet*	*Ryerson*	*cones*	*teddy*
10 / 28.14	GF+L0-Sm	31.27	32.01	31.13	29.69	29.38
	K-SVD	40.74	41.59	40.49	39.10	39.23
	BM3D	41.14	43.19	41.54	40.41	41.20
	prop.	**42.03**	**43.69**	**41.81**	**40.84**	**41.32**
30 / 18.59	GF+L0-Sm	30.65	31.10	29.99	28.99	28.15
	K-SVD	32.71	34.21	33.88	32.04	32.32
	BM3D	33.51	35.81	35.00	32.65	33.20
	prop.	**34.32**	**36.63**	**35.66**	**33.53**	**33.75**
50 / 14.16	GF+L0-Sm	29.53	30.41	29.02	28.06	27.46
	K-SVD	29.54	30.98	30.91	28.88	29.23
	BM3D	30.91	32.59	32.23	29.58	29.80
	prop.	**31.58**	**33.65**	**33.05**	**30.50**	**30.52**
100 / 8.14	GF+L0-Sm	26.76	27.99	26.36	25.57	25.26
	K-SVD	25.63	26.21	26.74	25.19	25.10
	BM3D	27.83	29.08	28.86	26.58	26.52
	prop.	**28.10**	**29.78**	**28.91**	**27.08**	**26.75**

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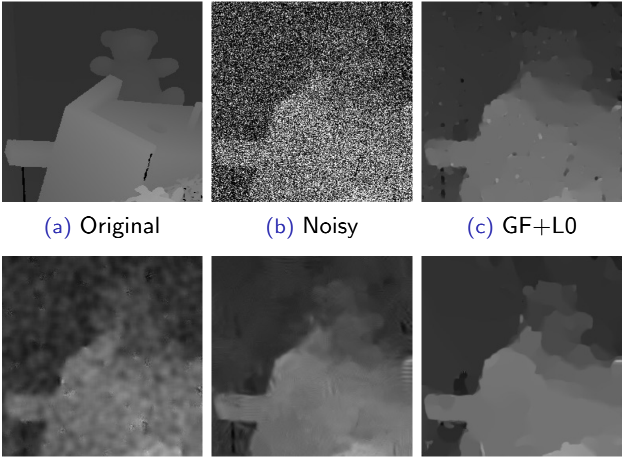
Experimental Results: Side-by-Side Comparisons

teddy, $\sigma = 100$



(a) Original (b) Noisy (c) GF+L0

(d) K-SVD (e) BM3D (f) CGWS

Figure: Denoising results.

Outline

Image Processing with Nonlocal Filters

- Many **nonlocal filters** and processing so far
  - **Bilateral filter** : Two Gaussian weights for distance and pixel values
  - Weights between i and j-th pixels:

$$w_{ij} = \exp\left(-\frac{||\mathbf{p}_i - \mathbf{p}_j||^2}{2\sigma_c^2}\right)\exp\left(-\frac{(x_i - x_j)^2}{2\sigma_s^2}\right)$$

  $\mathbf{p}_i$: Coordinate of i-th pixel, x_i: i-th pixel value, σ_c, σ_s: Std. dev. for Gaussian function

- **Trilateral filter** : Extension of bilateral filter
  - Gradient and pixel smoothing with BF
  - High smoothing performance (compared to bilateral filter)
  - Image filtering, tone-mapping of high dynamic range imaging

Problem of nonlocal filters

Pixel-dependent filtering: **No expression in frequency domain**

Bilateral Filter in Graph Spectral Domain

- Graph signal processing enables to represent **nonlocal filters on graph spectral domain** [Gadde et al. 2013].
- $\mathbf{x}$, $\widehat{\mathbf{x}}$: Vectorized input and output pixels
- $\mathbf{W} = [w_{ij}]$, $\mathbf{D}_{jj} = \sum_j w_{ij}$
- Graph bilateral filter:

$$\widehat{\mathbf{x}} = \mathbf{D}^{-1}\mathbf{W}\mathbf{x} = \underbrace{\mathbf{U}}_{\text{Inv. GFT}} \underbrace{(\mathbf{I} - \mathbf{\Lambda})}_{\text{Graph LPF}} \underbrace{\mathbf{U}^T}_{\text{GFT}} \mathbf{x}$$

$$\rightarrow h(\lambda) = 1 - \lambda$$

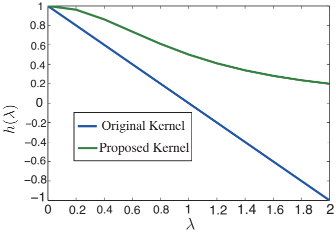
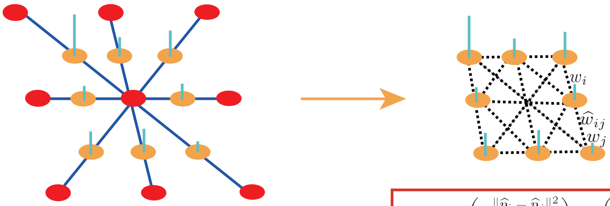


Figure: Kernels of graph bilateral filter

- **Graph filter kernels can be arbitrary changed according to applications.**

Graph Trilateral Filter

- Gradient smoothing can also be represented as graph spectral filters: **Double lowpass filters in graph spectral domain**
- Algorithm
  1. Calculate image gradient and construct **gradient graph**
  2. **Gradient smoothing** by graph bilateral filter
  3. construct image graph with smoothed gradient
  4. Pixel value smoothing by graph bilateral filter
- Both smoothing filters can be chosen arbitrary



$$\widehat{w}_{ij} = \exp\left(-\frac{\|\widehat{p}_i - \widehat{p}_j\|^2}{2\sigma_c^2}\right)\exp\left(-\frac{(w_i - w_j)^2}{2\sigma_s^2}\right)$$

Experimental Results

- AWGN denoising experiment
- Test images (128×128): *Lena*, *Watch*, *Boat*, and *Monarch*
- White gaussian noise : $\sigma = 20, 30, 40, 50$
- Comparison with regular/graph bilateral filter and regular trilateral filter

Lena

Watch

Boat

Monarch

Experimental Results: PSNR Comparisons

Table: Denoising Results: PSNR (dB)

Images	σ	50	40	30	20
Noisy	–	14.16	16.09	18.59	22.13
Lena	BF	15.71	17.99	21.34	26.21
	TF	16.09	18.63	21.27	25.59
	SGBF	16.11	18.57	22.06	26.69
	Proposed	**20.55**	**22.56**	**24.80**	**27.81**
Watch	BF	15.74	17.93	21.55	26.47
	TF	16.59	18.93	21.88	26.17
	SGBF	16.21	18.68	22.06	26.66
	Proposed	**20.63**	**22.36**	**24.59**	**27.65**
Boat	BF	16.06	18.27	21.39	26.27
	TF	16.60	18.54	21.98	25.52
	SGBF	16.32	18.58	21.97	26.45
	Proposed	**20.69**	**22.32**	**24.94**	**27.42**
Monarch	BF	15.83	18.03	21.12	26.01
	TF	16.55	18.53	20.91	24.54
	SGBF	16.04	18.48	21.83	26.11
	Proposed	**20.46**	**22.05**	**24.26**	**27.25**

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Experimental results: Side-by-Side Comparison

Lena, $\sigma = 30$


Original


Noisy


BF


TF


SGBF


Proposed

Conclusions

- **Oversampled graph transforms**
  - Graph oversampling with oversampled graph Laplacian matrix
  - Oversampled graph filter banks
  - Good for graph signal analysis and reasonable redundancy
- **Image processing**
  - Modification of BM3D with collaborative graph wavelet shrinkage
  - Image smoothing by graph-based trilateral filter

Further Reading

- Papers and Proceedings

  1. A. Sakiyama and Y. Tanaka, "Oversampled graph Laplacian matrix for graph filter banks," *IEEE Trans. Signal Process.*, 2014, in press. (Open Access)
  2. Y. Tanaka and A. Sakiyama, "*M*-channel oversampled graph filter banks," *IEEE Trans. Signal Process.*, vol. 62, no. 14, pp. 3578–3590, 2014. (Open Access)
  3. Y. Iizuka and Y. Tanaka, "Depth map denoising using collaborative graph wavelet shrinkage on connected image patches," *ICIP 2014*, Paris, France, Oct. 2014.
  4. M. Onuki and Y. Tanaka, "Trilateral filter on graph spectral domain," *ICIP 2014*, Paris, France, Oct. 2014.

- Review Article (**in Japanese!**)

  1. Y. Tanaka, "Recent Advances in Signal Processing on Graphs," *IEICE Fundamentals Review*, vol. 8, no. 1, pp. 15–29, 2014.

- MATLAB codes and slides
  - http://tanaka.msp-lab.org/software
  - http://tanaka.msp-lab.org/bio

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